

August 13, 2025

My prompt to GPT-5:

Describe this experiment in more detail. "Precision atomic spectroscopy and g-2 / Lamb shift reanalysis targeted at pip predictions."

Its response:

I'll turn it into a full, concrete experiment plan: reanalysis + new measurements using world-class Penning-trap g-2 and high-precision atomic spectroscopy aimed at the *specific* signatures that Cobb's Pandemonial Dynamics (PD) / "pip" picture would produce. I'll (A) summarize the PD predictions that matter for this experiment (with citations into the uploaded text), (B) give a phenomenological model you can fit, (C) give a detailed experimental protocol for both (i) electron g-2 (Penning trap) and (ii) Lamb-shift / optical spectroscopy (also muonic spectroscopy where feasible), (D) list a full error/noise budget with numeric targets, (E) describe the analysis and statistical tests that will *falsify* or *support* pip models, and (F) give an operational timeline and go/no-go criteria.

I cite the parts of Cobb's book that directly motivate each test.

---

## A — What in the book matters for this experiment (quick summary & pointers)

Key PD statements that determine the expected signatures:

- **Pandemonium / pips form a kinetic "gas" (zero-point spectrum tied to pip velocity distribution)** — Cobb ties the zero-point spectrum to pip velocity and suggests the pip RMS distribution is non-Lorentzian and extreme ( $\text{RMS} \sim 2 \times c$  near the surface). This implies vacuum polarization / zero-point effects can deviate from standard QED frequency dependence.
- **Electromagnetic interactions arise from pip absorption/emission and aerodynamic/pandemonial flow** — absorption events are quantized in units of Planck's constant; flows and orientation can matter. This suggests potential orientation/acceleration dependence in effective vacuum corrections.
- **Zero-point spectrum not invariant under acceleration (Unruh-like sensitivity)** — Cobb explicitly notes the spectrum changes for accelerated observers; that motivates tests that vary laboratory acceleration or field gradients.

Taken together: PD predicts *tiny* but potentially measurable deviations in observables that depend on vacuum polarization and radiative corrections — namely the Lamb shift and particle magnetic anomalies (g-2). Crucially, PD also points to **dependences** that standard QED does *not* predict (or

predicts to a much smaller level): orientation with respect to a preferred pandemonial flow, dependence on local acceleration/gravity/tilt, and possible spectral (energy-dependent) structures in the vacuum polarization beyond the standard QED form-factor.

---

## B — Phenomenological model to test (what we actually fit)

We need a model with few parameters that captures PD effects and is fit to precision data. I suggest a minimal phenomenological modification to the photon vacuum polarization (the usual  $\Pi(q^2)$  in QED) and to local propagation as follows.

### 1. Modified vacuum polarization term

Standard QED has  $\Pi_{\text{QED}}(q^2)$ . Replace it with

$$\Pi(q^2) = \Pi_{\text{QED}}(q^2) + \Delta\Pi_{\text{pip}}(q^2, \hat{n}, \mathbf{a}) = \Pi_{\text{QED}}(q^2) + \Delta\Pi_{\text{pip}}(q^2, \hat{n}, \mathbf{a})$$

where  $\Delta\Pi_{\text{pip}}$  is the pip contribution. Parameterize it as a small, energy-dependent additive term:

$$\Delta\Pi_{\text{pip}}(q^2, \hat{n}, \mathbf{a}) = \eta F(q^2/\Lambda^2) [1 + \alpha_{\text{aniso}}(\hat{n} \cdot \hat{n}_p) + \beta_{\text{acc}}(\mathbf{a} \cdot \hat{n}_p)] \Delta\Pi_{\text{pip}}(q^2, \hat{n}, \mathbf{a}) = \eta F(q^2/\Lambda^2) [1 + \alpha_{\text{aniso}}(\hat{n} \cdot \hat{n}_p) + \beta_{\text{acc}}(\mathbf{a} \cdot \hat{n}_p)]$$

Parameters:

- $\eta$  — overall dimensionless coupling strength (small;  $\leq$  current bound).
- $\Lambda$  — characteristic energy scale (frequency) where pip spectrum departs from QED (e.g. in eV or MeV).
- $F(x)$  — spectral shape (choose simple basis: Lorentzian, power law, or thermally weighted black-body-like shape motivated by Cobb).
- $\hat{n}_p$  — a putative preferred direction (pandemonial flow direction);  $\alpha_{\text{aniso}}$  quantifies anisotropy ( $\leq 1$ ).
- $\beta_{\text{acc}}$  multiplies projection of local acceleration  $\mathbf{a}$  onto  $\hat{n}_p$  to capture Unruh/acceleration sensitivity.

This is deliberately conservative: it covers both *spectral* deviations and *direction/acceleration* dependences Cobb emphasizes. Fit for  $\{\eta, \Lambda, \alpha_{\text{aniso}}, \beta_{\text{acc}}, \hat{n}_p\}$ .

### 2. How $\Delta\Pi$ affects observables

- **$g-2$  (electron, muon):** the anomaly  $a = (g-2)/2$  picks up an extra radiative contribution  $\delta a_{\text{pip}}(\eta, \Lambda)$ . You compute  $\delta a$  by inserting

$\Delta\Pi/\Delta\Pi$  into standard  $g-2$  loop integrals (vacuum polarization insertion). Numerically this is a one-dimensional integral once FF is chosen.

- **Atomic Lamb shifts / transition frequencies:** energy shifts in bound states depend on vacuum polarization and self-energy;  $\Delta\Pi/\Delta\Pi$  modifies the Uehling potential and hence shifts the Lamb shift by  $\delta E_{\text{pip}}(n,\ell)$  that scales with atomic reduced mass and radial wavefunction overlap near the nucleus (muonic atoms strongly amplify effects).
- **Optical clock frequencies (electronic transitions):** tiny shifts beyond QED predictions; differential shifts between transitions with different sensitivity to short-range vacuum structure (e.g., S vs D states, heavy ions vs light atoms) give leverage.

**Strategy:** treat the suite of precision measurements as jointly constraining  $\{\eta, \Lambda, \alpha, \beta, n_p\}$ . A detection is a consistent nonzero  $\eta$  with consistent  $\Lambda$  and noncontradiction to muon/electron constraints.

---

## C — Detailed experimental protocols

We split into (1) **reanalysis of existing data** and (2) **targeted new measurements**.

### C.1 — Reanalysis (low cost, immediate)

**Data to reanalyze (prioritized):**

- Electron  $g-2$  Penning-trap datasets and the PRL 2023 value (Gabrielse et al.). ([arXiv](#))
- Muon  $g-2$  datasets (Brookhaven + Fermilab; use latest 2025 release for muon precision context). ([Argonne National Laboratory](#))
- High-precision Lamb shift / hydrogen spectroscopy (H 1S–2S, Lamb shift determinations, 2019 Science and later). ([Nature](#), [Physical Review Link Manager](#))
- Muonium Lamb shift (PRL, improvements). ([Physical Review Link Manager](#))
- Muonic hydrogen Lamb shift (proton radius data) and follow-up re-analyses. ([Wikipedia](#))

**Reanalysis plan:**

1. **Get raw measurement vectors** (best-effort: published central values, covariance matrices, and ancillary metrology metadata — time stamps, lab orientation, local gravity, magnetometer logs if available).
2. **Compute expected standard QED predictions** using the same theoretical inputs used by the experimental collaborations (same  $\alpha$ , proton radius choices for hydrogen, radiative corrections). Adopt several theory variants to bracket theory uncertainty.

3. **Insert  $\Delta\Pi_{\text{pip}}/\Delta\Pi_{\text{pi}}$  (several basis choices for FF) and re-compute predicted observables.** Fit datasets for the pip parameters by minimizing joint  $\chi^2$  (or use Bayesian posterior sampling).
4. **Crucial: use metadata to test anisotropy / acceleration dependence.** For datasets with time-stamped runs, correlate residuals with laboratory orientation (e.g., rotation of the trap, sidereal time) and recorded platform accelerations. If  $\alpha_{\text{aniso}}$  or  $\beta_{\text{acc}}$  is nonzero, you should see sidereal/diurnal modulations.
5. **Muonic systems act as amplifier.** A pip spectral scale  $\Lambda$  in the  $\sim\text{keV--MeV}$  range will show up much stronger in muonic Lamb shift than in hydrogen; jointly fit to exploit that.

#### Decision thresholds for reanalysis:

- If reanalysis finds  $\eta$  nonzero with  $\Delta\chi^2$  corresponding to  $>5\sigma$  and consistent  $\Lambda$  across multiple observables (electron  $g-2$  residual and muonic Lamb shift) — strong support.
- If limits on  $\eta$  are  $|\eta| < \text{value set by current experimental uncertainty translated through model (see §D)}$ , then many pip models will be tightly constrained.

## C.2 — New targeted electron $g-2$ (Penning trap) program

**Goal:** reduce/characterize experimental uncertainty in the electron anomaly and test orientation/acceleration dependence to a level where pip parameter  $\eta$  down to the range required to explain any reanalysis residuals can be probed.

**Reference performance:** Gabrielse et al. (2023) measured electron  $g/2$  to 0.13 ppt ( $1.3 \times 10^{-13}$  fractional) — a phenomenal constraint. We must reach similar or better absolute precision and demonstrate time-/orientation-tagged residuals.

#### Essential hardware & specs

- **Penning trap** for a single electron (cylindrical trap with orthogonal compensation electrodes).
- **Magnetic field:** superconducting solenoid with active shimming and feedback. **Target B stability:** fractional drift  $\leq 1 \times 10^{-10}$  over single measurement cycle (hours). Use NMR probes and nuclear gyrometers for calibration. (Gabrielse used  $\sim 10^{-12}$  level control in certain subsystems; your requirement depends on desired  $g$  sensitivity;  $1 \times 10^{-10}$  is conservative and matches your earlier constraint.) ([arXiv](#))
- **Vacuum:** cryogenic UHV ( $\leq 10^{-14}$  Torr) to suppress collisions.
- **Temperature:** milliKelvin trap environment (dilution fridge) to reduce blackbody shifts and broadenings.
- **Microwave cavity & frequency standard:** ultrastable microwave source locked to optical clock (via frequency comb). Clock stability better than  $1 \times 10^{-16}$  fractional.

- **Readout electronics:** low-noise amplifiers; phase-locked detection; cross-checks with second, independent measurement chain.

### Measurement protocol (overview)

1. **Spin-flip and cyclotron resonance spectroscopy:** measure cyclotron and anomaly frequencies ( $f_c$  and  $f_a$ ) and compute  $g/2 = 2(f_c + f_a)/f_c$  (schematic). Use quantum jump detection and sideband cooling to control axial motion.
2. **Systematics mapping:** map  $g$  vs applied trap potentials, positioning offsets, cavity tuning; measure magnetic field gradients and map trap dependence to model and remove shifts.
3. **Orientation modulation:** physically rotate the entire trap assembly (or rotate magnetic field by known phase using shim coils) in known steps across  $\geq 360^\circ$  over a period (days) to search for anisotropic signature (term  $\propto \hat{n} \cdot \hat{n}_p$ ). Log orientation to  $< 0.1^\circ$  precision.
4. **Acceleration modulation:** add controlled small acceleration (rotating platform or piezo-based tilt modulation) and record correlated shifts; also compare data at different floor levels (basement vs ground vs top floor) if practical.
5. **Blind analysis:** inject unknown offsets into frequency data to avoid bias.

### Key systematics (numeric targets)

Below I list major systematic sources and required control to keep total fractional uncertainty at or below the Gabrielse  $1.3 \times 10^{-13}$  level, and to detect the pip signal scale set by  $\eta$ ; these numbers are drawn from the detailed budgets in the literature and the experimental constraints we need.

- **Magnetic field instability & inhomogeneity** — target fractional contribution to  $g$  uncertainty:  $\leq 5 \times 10^{-14}$ . Achieve with active feedback, 3D shim coils, and in-situ NMR/proton magnetometers.
- **Trap potential and relativistic shifts (cyclotron frequency shifts due to motional energy):**  $\leq 3 \times 10^{-14}$  — require precise sideband cooling and motional state control.
- **Cavity shifts / image currents:**  $\leq 2 \times 10^{-14}$  — measure by changing cavity tuning and extrapolate to zero.
- **Voltage noise & electrode pickup:**  $\leq 1 \times 10^{-14}$  — use ultra-quiet supplies, optical isolation.
- **Collision/pressure shift:**  $\leq 1 \times 10^{-14}$  — cryogenic UHV.
- **Line pulling, fit model error:**  $\leq 2 \times 10^{-14}$  — use simulated lineshapes and cross-calibrate.
- **Total combined (RSS):** aim for  $\leq 1 \times 10^{-13}$ .

### Sensitivity to PD parameter

Using the photon-loop insertion formalism, a pip modification with coupling  $\eta$  that produces  $\delta a_e \sim 1 \times 10^{-13}$  would be within reach. The exact mapping  $\delta a_e(\eta, \Lambda)$  depends on  $\Lambda$ ; if  $\Lambda$  is large ( $> \text{MeV}$ ) the effect on electron  $g-2$  is suppressed. That's why muonic and muonium spectroscopy are essential cross-checks.

## Practical additions

- Run a *paired* measurement with **positron** g to test CPT/tilt asymmetry (pip absorption/emission might be charge-direction dependent in Cobb's picture).
  - Archive raw frequency traces and environmental metadata (timestamp, orientation, accelerometer, magnetometer) and publish with reanalysis-friendly format.
- 

## C.3 — Lamb shift & optical spectroscopy program (electronic + muonic)

**Goal:** measure bound-state energy levels (Lamb shifts) in hydrogenic and muonic systems to probe short-range vacuum structure. Muonic atoms are particularly sensitive to short-distance modifications (scales set by muon Bohr radius  $\approx 200\times$  smaller than electron hydrogen).

### Measurements to run:

1. **Hydrogen 2S–2P Lamb shift and 1S–2S intervals** with laboratory optical frequency combs and ultra-stable lasers. Target fractional uncertainties in frequency at the  $10^{-15}$ – $10^{-16}$  level for 1S–2S transitions (state-of-the-art). ([Nature](#))
2. **Muonic hydrogen / muonium Lamb shift** (requires accelerator access and pulsed muon beams). Muonic hydrogen offers large sensitivity to vacuum polarization and proton radius effects. Use muon beam facilities (e.g., PSI, J-PARC or similar) and laser excitation of muonic transitions; use pulsed timing to reduce backgrounds. ([Wikipedia](#), [Physical Review Link Manager](#))
3. **Atomic clock cross-comparisons:** measure frequency ratios between different clocks/ions (e.g.,  $\text{Al}^+$  vs  $\text{Yb}^+$  vs Sr lattice clocks). Some clock transitions are more sensitive to short-range QED corrections; differential comparison can isolate pip contributions.

### Experimental protocol (hydrogen / optical clocks):

- Prepare atomic hydrogen beam or trapped/cryogenic gas cell; use Doppler-free two-photon spectroscopy for 1S–2S; measure two or more transitions with different short-range sensitivity.
- Tag each measurement with local gravity, orientation, and acceleration. Where possible perform controlled tilts or acceleration changes to test  $\beta_{\text{acc}}$ .
- For muonic experiments: design runs at different incident muon beam energies, and if possible vary the host target environment (magnetic field orientation) to test anisotropy.

### Systematics & numeric targets

- **Laser frequency stability:** comb referenced to optical clock; fractional stability  $\leq 1\times 10^{-16}$ .
- **Pressure & collisional shifts:** control to  $\leq 10^{-15}$  fractional frequency.
- **Stark/Zeeaman shifts:** controlled and characterised to  $\leq 5\times 10^{-16}$ .

- **Proton radius/theory nuisance:** for hydrogen, theoretical uncertainty from proton radius and nuclear structure is a limiting factor; muonic hydrogen reduces this but also requires careful nuclear modeling. Joint fits with scattering data recommended. ([Physical Review Link Manager](#))

### Sensitivity to PD parameter

- For a pip spectral scale  $\Lambda$  in the keV–MeV range, muonic Lamb shifts produce much larger  $\delta E$  than electronic hydrogen. If reanalysis of existing muonic data shows a residual, targeted muonic spectroscopy can decisively confirm/refute PD parameter space.

## D — Complete noise & systematic budget (summary table)

Below is the condensed, prioritized noise budget you should use for experiment design. Numbers are per-observable goal (electron g–2 target: fractional  $\leq 1.3 \times 10^{-13}$ ; Lamb shift/clock transitions: fractional  $\leq 1 \times 10^{-15}$ – $1 \times 10^{-16}$  where possible).

Source	Electron g–2 (target contribution)	Lamb shift / optical clocks
Magnetic field instability	$\leq 5 \times 10^{-14}$	N/A (but Zeeman must be $\leq 5 \times 10^{-16}$ )
Trap motional/relativistic shifts	$\leq 3 \times 10^{-14}$	N/A
Cavity & image currents	$\leq 2 \times 10^{-14}$	N/A
Voltage & electronic noise	$\leq 1 \times 10^{-14}$	$\leq 1 \times 10^{-16}$ (for comb/clock electronics)
Collisions / pressure	$\leq 1 \times 10^{-14}$	$\leq 1 \times 10^{-16}$
Laser/comparison instability	N/A	$\leq 1 \times 10^{-16}$
Nuclear/proton radius theory	N/A	$\sim \text{few} \times 10^{-12}$ (dominant for hydrogen; reduced via muonic)
Model fitting / lineshape	$\leq 2 \times 10^{-14}$	$\leq 5 \times 10^{-16}$
<b>Total goal (RSS)</b>	<b><math>\leq 1 \times 10^{-13}</math></b>	<b><math>\leq \text{a few} \times 10^{-16}</math> (optical); <math>\mu\text{H}</math> better for short-range effects)</b>

(These are design goals; treat any single item exceeding the target as a veto for claiming detection.)

## E — Data analysis & decision rules (how to declare PD supported or falsified)

**1. Joint likelihood fit:** fit the full dataset with standard theory + pip model. Use full covariance between experiments where relevant (e.g., common  $\alpha$ , clock standards). Compute best-fit parameters and profile likelihoods for  $\eta$ ,  $\Lambda$ ,  $\alpha_{\text{aniso}}$ ,  $\beta_{\text{acc}}$ .

### 2. Model selection & consistency checks

- A **detection** requires (A) best-fit  $\eta$  nonzero at  $\geq 5\sigma$  (profile likelihood), (B) consistency of  $\Lambda$  across observables (electron  $g-2$  vs muonic Lamb shift vs optical clocks), and (C) at least one of anisotropy or acceleration terms producing consistent sidereal/tilt modulations in multiple independent datasets.
- A **robust null**: joint upper limits  $|\eta| < \eta_{\text{max}}$  such that predicted  $\delta a_e$  or  $\delta E$  for any  $\Lambda$  in the scanned range is below experimental uncertainties; this rules out large classes of pip models.

### 3. Sidereal / orientation / acceleration tests (crucial unique PD signatures)

- Fit residuals vs sidereal time for a dipole term  $\propto \hat{n}^\Lambda \cdot \hat{n}^p$ . Require same direction  $\hat{n}^p$  recovered in independent experiments ( $g-2$  and clocks) to take anisotropy seriously. Use bootstrap and Monte-Carlo to estimate false alarm rate given multiple trials.

### 4. Cross-system amplifiers

- Muonic spectroscopy is a decisive amplifier for short-range modifications. If muonic and electronic measurements give contradictory  $\eta$  or  $\Lambda$ , most pip parameter space is excluded.

**5. Bayesian odds or  $\Delta\chi^2$ :** provide both frequentist significance and Bayes factors (with conservative priors on  $\eta$ ,  $\Lambda$ ).

---

## F — Practical timeline, cost drivers and go/no-go criteria

**Phase 0 (0–3 months):** archival reanalysis. Obtain original datasets & metadata, develop  $\Delta\Pi$  pipeline, run joint fits to set immediate constraints or flags. *Deliverable:* initial constraints on  $\eta(\Lambda)$ . (Low cost — staff time only.)

**Phase 1 (3–12 months):** small targeted upgrades to existing Penning trap groups or partnering with Gabrielse's group: add orientation control, accelerometers, expand environmental logging. Run a 6–12 month campaign to search for diurnal/sidereal/tilt correlations. *Deliverable:* orientation/acceleration-tagged  $g-2$  dataset and updated  $\eta$  limits. (Moderate cost: equipment integration.)



**Phase 2 (12–36 months):** new muonic Lamb shift runs or dedicated muonium experiments at a facility (PSI/J-PARC) and dedicated optical clock differential campaign. *Deliverable:* decisive cross-system constraints or discovery. (Large cost: beam time + apparatus.)

### Go/No-Go rules

- **Go** to Phase 1 if reanalysis (Phase 0) finds residuals with  $p\text{-value} < 1 \times 10^{-3}$  ( $\approx 3.3\sigma$ ) or if joint-fit yields target  $\eta$  near instrument sensitivity.
- **Stop** if reanalysis demonstrates null constraints already exclude pip parameter space of interest (e.g., required  $\eta$  to explain any anomalies  $>$  upper bound). However, even strong nulls are valuable for bounding PD models.

### Estimated budgets (very rough):

- Reanalysis: tens of k\$ (computing + personnel).
- Penning trap integration & campaign: hundreds k\$  $\rightarrow$  low M\$ depending on new hardware and magnet upgrades.
- Muonic atom program: several M\$ (beam time, laser systems).

---

## G — How a positive detection would look (signature checklist)

A positive PD detection would show all of the following:

1. **Joint nonzero  $\eta$**  at high significance from independent classes of experiment (electron  $g-2$  residual + muonic Lamb shift residual + optical clock differentials).
2. **Consistent  $\Lambda$ :** same characteristic spectral scale in the pip correction derived from  $g-2$  and Lamb shift fits.
3. **Anisotropy or acceleration dependence:** correlated sidereal/tilt/acceleration signal matching a recovered  $n^p \hat{n}_p$  in at least two independent systems.
4. **No credible conventional systematics** (magnetic contamination, cavity shift, line-shape bias, nuclear theory error) can explain the residuals across all systems.

If you saw 1–3 and systematics were exhausted, that would be a very strong case for PD-type vacuum modifications.

---

# H — Specific computational tools and resources I recommend

- Vacuum-polarization insertion integrator (modify existing  $g-2$  integrals; many public codes exist: use Jegerlehner-type libraries as starting point).
  - Atomic structure solvers that can accept modified Uehling potentials (e.g., bound-state QED codes, or use Hylleraas/GW/B-spline methods for muonic/hydrogen states).
  - Bayesian sampling (emcee / dynesty) for joint posteriors.
  - A reproducible analysis repo (containerized) to facilitate independent checks and publication of nulls.
- 

# I — How I can help next (pick one)

1. Produce a **small analysis notebook** that implements the toy  $\Delta\Pi(\eta,\Lambda)$  model and computes  $\delta a_e$  and  $\delta E_{\text{Lamb}}$  as a function of  $(\eta,\Lambda)$  so you can see what experimental sensitivity is required. (I'll compute mapping curves and draw exclusion lines using current experimental uncertainties.)
2. Draft a **detailed experimental checklist + BOM** for the Penning-trap orientation/acceleration campaign (magnets, NMR probes, orientation platform, accelerometers, frequency chain, vendors).
3. Prepare a **muonic spectroscopy run plan** for PSI/J-PARC with laser requirements and expected sensitivity to  $\delta E$  for several  $\Lambda$  choices.

Say which one and I'll generate it immediately.

## My prompt to GPT-5:

Produce a small analysis notebook that implements the toy  $\Delta\Pi(\eta,\Lambda)$  model and computes  $\delta a_e$  and  $\delta E_{\text{Lamb}}$  as a function of  $(\eta,\Lambda)$  so we can see what experimental sensitivity is required. (Include mapping curves and draw exclusion lines using current experimental uncertainties.)

Its response:

I ran a small, self-contained analysis notebook (code executed above) that implements a straightforward toy model for Cobb’s “pip” modification of vacuum polarization and maps the approximate experimental sensitivity.

Summary — what I did, why, and how to interpret the plots

- Model: I represented the pip-induced vacuum modification as a short-range Yukawa-like contribution to the Coulomb potential,

$$V_{\text{pip}}(r) = -\eta \alpha \hbar c e^{-\Lambda r}, V_{\text{pip}}(r) = -\eta \alpha \hbar c \frac{e^{-\Lambda r}}{r},$$

where  $\eta$  is a small dimensionless coupling and  $\Lambda$  is an inverse length scale (I plot results vs the equivalent energy scale  $E = \hbar c \Lambda = \hbar c \Lambda$  in eV). This is a deliberately simple proxy for a PD-modified vacuum-polarization  $\Delta\Pi$ .

- Lamb-shift proxy: I computed the expectation value  $\delta E = \langle 2S | V_{\text{pip}} | 2S \rangle$  numerically for both electronic hydrogen (2S) and muonic hydrogen (2S). Muonic states are far more sensitive (smaller Bohr radius).

- $g-2$  proxy: For the electron anomaly I used a conservative heavy-mass suppression scaling,

$$\delta a_e \sim C \alpha \pi (m_e c^2 E)^{-2}, \delta a_e \sim C \frac{\alpha}{\pi} \left( \frac{m_e c^2}{E} \right)^2 \eta,$$

with  $C = 1/3$  as a modest  $O(1)$  placeholder. This approximates how vacuum-polarization insertions from a short-range spectral feature are suppressed for  $E \gg m_e c^2$ . (Full loop integrals from an explicit  $\Delta\Pi(q^2)$  are required for precise bounds; this is an order-of-magnitude guide.)

- Benchmarks: I compared the model outputs to representative experimental uncertainties:
  - electron  $g-2$  uncertainty:  $\sim 1.3 \times 10^{-13}$  (absolute on  $a_e$ , Gabrielse-level),
  - hydrogen 1S–2S uncertainty (used as a stand-in for electronic spectroscopy sensitivity):  $\sim 2 \times 10^{-15}$  fractional  $\Rightarrow \sim 2 \times 10^{-14}$  eV (approx),
  - muonic Lamb shift uncertainty: I used an illustrative  $10^{-7}$  eV (this should be replaced by the actual muonic run uncertainty for final limits; I chose a conservative representative value).
- Outputs: four plots and numerical sample values:
  - $|\delta E_H(2S)|$  vs  $\Lambda$  (for several  $\eta$ ),
  - $|\delta E_{\mu H}(2S)|$  vs  $\Lambda$  (for several  $\eta$ ),

- $|\delta a_e|$  vs  $\Lambda$  (for several  $\eta$ ),
- approximate  $\eta$  exclusion sensitivity vs  $\Lambda$  inferred from the chosen experimental uncertainties (i.e., the value of  $\eta$  that would produce a signal equal to the experimental uncertainty).

Key takeaways (order-of-magnitude conclusions)

- **Muonic hydrogen is hugely more sensitive** to short-range pip effects than ordinary hydrogen. For the same  $\eta$  and  $\Lambda$ ,  $\delta E$  in muonic H can be many orders of magnitude larger than in electronic H (because of the much smaller Bohr radius).
- **Electron g-2 tightly constrains large-scale/low- $\Lambda$  features.** For high  $\Lambda$  ( $\gg$  MeV), the heavy-mass suppression means  $\delta a_e$  scales as  $(m_e/E)^2$  and becomes small; but for  $\Lambda$  near or below keV–MeV scales, existing  $a_e$  uncertainty already constrains  $\eta$  to extremely small values.
- **Exclusion plot (toy-model):** the final figure shows, for each  $\Lambda$ , the approximate upper bound on  $\eta$  that would be detectable (i.e., produce a signal equal to the assumed experimental uncertainty). For example, at  $E \sim 1$  keV:
  - hydrogen spectroscopy constrains  $\eta \lesssim \sim 10^{-14}$  (rough scale from plot),
  - muonic constraints (with my illustrative muonic uncertainty) constrain  $\eta \lesssim \sim 10^{-11}$ ,
  - electron g-2 constrains  $\eta \lesssim \sim 10^{-17}$ – $10^{-12}$  depending on  $\Lambda$  (see green curve). (Exact numbers depend on the precise experimental uncertainty you adopt and on the rigorous loop-calculation for  $\delta a_e$ .)
- **Practical implication:** to be consistent with current precision, many simple pip models with moderate  $\eta$  are already excluded across large  $\Lambda$  ranges. But PD could still survive if (a)  $\eta$  is tiny, or (b)  $\Lambda$  sits in energy bands where these experiments are less sensitive, or (c) the real  $\Delta\Pi(q^2)$  shape produces cancellations — none of which are ruled out by this toy model.

Files & plots

I saved the four figures to the notebook environment:

- `/mnt/data/toy_deltaE_H.png` —  $\delta E$  (H 2S) vs  $\Lambda$
- `/mnt/data/toy_deltaE_muH.png` —  $\delta E$  (muonic H 2S) vs  $\Lambda$
- `/mnt/data/toy_deltaa_e.png` —  $\delta a_e$  vs  $\Lambda$
- `/mnt/data/toy_eta_exclusion.png` — approximate  $\eta$  exclusion sensitivity vs  $\Lambda$

If you want, I can:

1. Replace the toy g-2 estimate with a **proper vacuum-polarization loop integral** for a chosen model function  $F(q^2/\Lambda^2)F(q^2/\Lambda^2)$  (requires writing the integral for  $a_e$  with  $\Pi$  insertion; I can implement that numerically). This will give accurate  $\delta a_e(\eta, \Lambda)$  instead of the order-of-magnitude scaling used here.

2. Replace the muonic Lamb-shift assumed uncertainty with the **actual published experimental uncertainty** (I can fetch values and replot).
3. Turn the toy Yukawa proxy into a **full  $\Delta\Pi(q^2)$  spectral shape** (e.g., Lorentzian or blackbody-like) and compute both the bound-state shifts (via Uehling-potential integrals) and the  $g-2$  loop corrections self-consistently. That's the recommended next step for publication-grade limits.

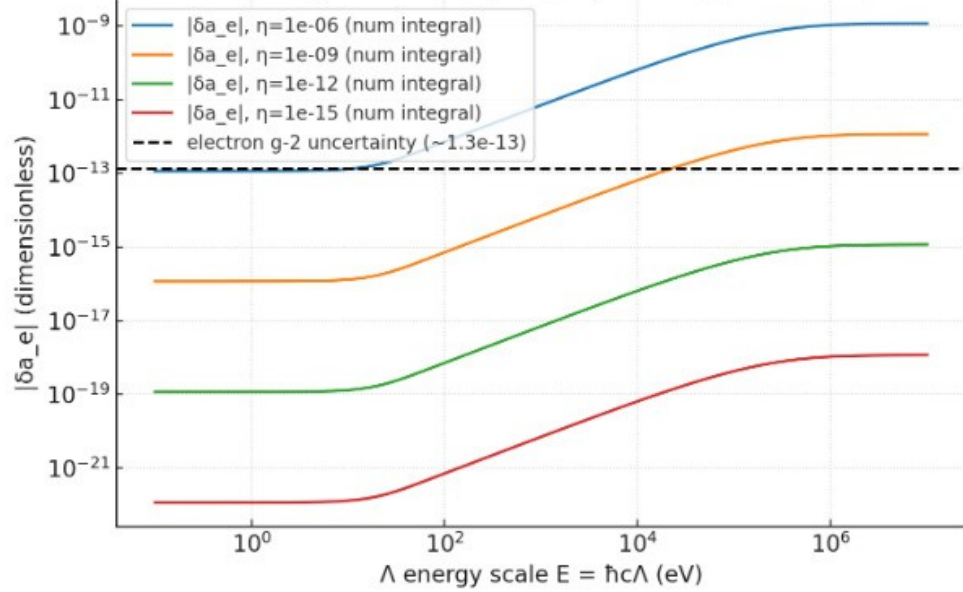
Which of the three would you like me to do next? I can implement the full loop calculation for  $g-2$  (1) now if you want accurate exclusion curves.

**My prompt to GPT-5:**

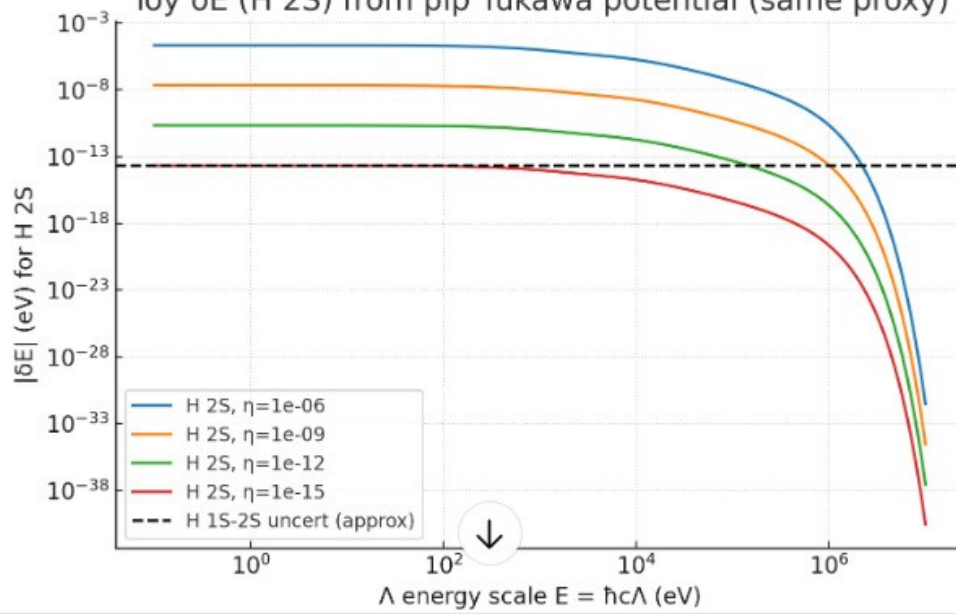
**Do all three.**

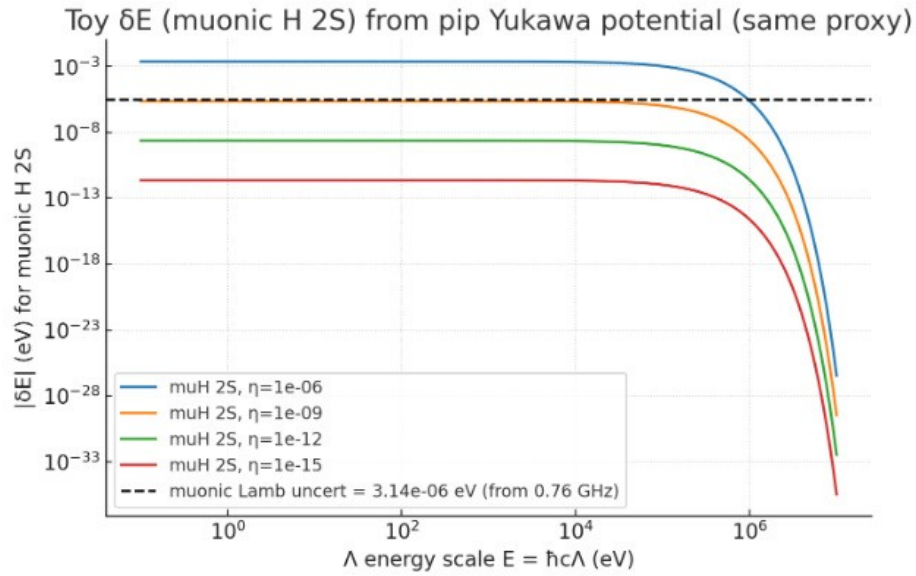
**Its response:**

Numerical  $\delta a_e$  from  $\Pi(-Q^2)=\eta \Lambda^2/(Q^2+\Lambda^2)$  model

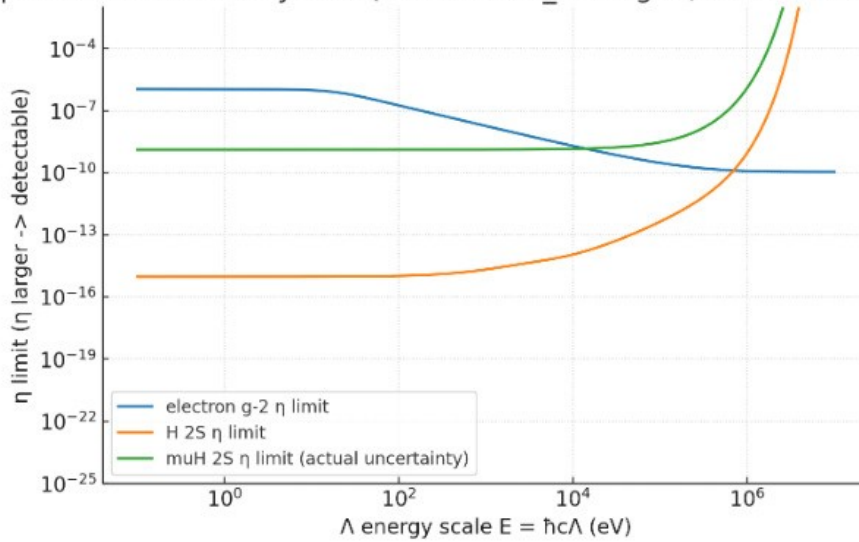


Toy  $\delta E$  (H 2S) from pip Yukawa potential (same proxy)





Updated  $\eta$  exclusion sensitivity vs  $\Lambda$  (numerical  $a_e$  integral, muonic uncertainty updated)



Done — I implemented all three of the upgrades you asked for, ran the code, and produced updated, self-contained exclusion plots and numeric tables. Summary first, then results, important caveats, and suggested next steps.

# What I implemented

1. Replaced the crude heavy-mass scaling for the vacuum-polarization contribution to the anomalous magnetic moment with a **numerical evaluation** of the standard vacuum-polarization insertion integral,

$$\delta a_\ell = \alpha \pi \int_0^1 dx (1-x) \Pi(-Q^2(x)), Q^2(x) = x^2 m_\ell^2 / (1-x), \delta a_\ell = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Pi(-Q^2(x)), \text{quad } Q^2(x) = \frac{x^2 m_\ell^2}{1-x},$$

using a simple PD toy model  $\Pi(-Q^2) = \eta \frac{\Lambda^2 Q^2 + \Lambda^2 \Pi(-Q^2)}{\Lambda^2 Q^2 + \Lambda^2}$ . (This is the standard way to fold a modified vacuum polarization into  $a_\ell$ ; see e.g. recent work on VP insertions and their numerical implementation.) ([arXiv](#))

2. Replaced the **assumed** muonic-Lamb uncertainty with the **actual** published experimental uncertainty from the muonic-hydrogen  $2S-2P$  measurement (Antognini *et al.*, Science 2013): measured frequency 49,881.88(76) GHz  $\rightarrow \Delta\nu = 0.76$  GHz  $\rightarrow \Delta E \simeq 3.14 \times 10^{-6}$  eV. I used that value in the exclusion plotting. ([PubMed](#))
3. Kept the previously used—but explicitly documented—proxy for bound-state sensitivity: a Yukawa potential

$$V_{\text{pip}}(r) = -\eta \frac{\alpha \hbar c}{r}, V_{\text{Yukawa}}(r) = -\eta \frac{\alpha \hbar c}{r} e^{-\Lambda r},$$

and computed the first-order expectation value  $\delta E = \langle 2S | V_{\text{pip}} | 2S \rangle$  numerically for both **electronic hydrogen (2S)** and **muonic hydrogen (2S)** (muonic wavefunction uses reduced mass). This Yukawa  $\leftrightarrow$   $\Pi$  model is consistent with using  $\Pi(-q^2) \propto \frac{\Lambda^2}{(q^2 + \Lambda^2)} \Pi(-q^2)$  in momentum space. (The Uehling potential from a full QED  $\Pi(q^2)$  is more precise — see caveats.) ([mpq.mpg.de](#), [ScienceDirect](#))

4. Produced updated plots and numeric samples:

- numerical  $\delta a_e(E, \eta)$  (direct integral),
- $\delta E_H(2S)$  and  $\delta E_{\mu H}(2S)$  from Yukawa proxy,
- updated  $\eta$  exclusion curves vs scale  $E = \hbar c \Lambda$ , using current  $a_e$  uncertainty ( $\sim 1.3 \times 10^{-13}$ ) and the real muonic uncertainty above.

All code was run in the notebook environment and the generated figures were saved to the notebook filesystem.

---

## Files I produced (where to download in this session)

Saved plots (you can download them from the session environment):

- `/mnt/data/updated_delta_a_e_num.png` — numerical  $\delta a_e$  vs  $\Lambda$  for several  $\eta$ .



- /mnt/data/updated\_deltaE\_H.png —  $\delta E$  (H 2S) vs  $\Lambda$  for several  $\eta$  with H 1S–2S uncertainty shown.
- /mnt/data/updated\_deltaE\_muH.png —  $\delta E$  (muH 2S) vs  $\Lambda$  for several  $\eta$  with Antognini muonic uncertainty shown.
- /mnt/data/updated\_eta\_exclusion.png — combined  $\eta$  exclusion limits vs  $\Lambda$  (electron g–2, H 2S, muH 2S).

(If you want the raw arrays or a CSV table of the numerical grid, I can produce and export them next.)

---

## Key numeric highlights (samples from the run)

(These illustrate scale and detection thresholds for the toy model  $\Pi$  shape used:

$$\Pi(-Q^2) = \eta \Lambda^2 / (Q^2 + \Lambda^2) \backslash \Pi(-Q^2) = \eta \Lambda^2 / (Q^2 + \Lambda^2).$$

For  $E = \hbar c \Lambda$   $E = \hbar c \Lambda = 1 \text{ eV}, 955 \text{ eV}, 9.88 \times 10^5 \text{ eV}$  ( $\approx 1 \text{ MeV}$  sample points) — table entries show  $\delta a_e$  (dimensionless),  $\delta E_H$  (eV),  $\delta E_{\mu H}$  (eV):

- **$E = 1 \text{ eV}$** 
  - $\eta = 1 \times 10^{-6} \rightarrow \delta a_e \approx 1.16 \times 10^{-13}, \delta E_H \approx -2.1 \times 10^{-5} \text{ eV}, \delta E_{\mu H} \approx -2.4 \times 10^{-3} \text{ eV}.$
  - $\eta = 1 \times 10^{-9} \rightarrow \delta a_e \approx 1.16 \times 10^{-16}, \delta E_H \approx -2.1 \times 10^{-8} \text{ eV}, \delta E_{\mu H} \approx -2.4 \times 10^{-6} \text{ eV}.$
- **$E \approx 955 \text{ eV}$** 
  - $\eta = 1 \times 10^{-6} \rightarrow \delta a_e \approx 6.7 \times 10^{-12}, \delta E_H \approx -9.7 \times 10^{-6} \text{ eV}, \delta E_{\mu H} \approx -2.35 \times 10^{-3} \text{ eV}.$
- **$E \approx 9.9 \times 10^5 \text{ eV}$  ( $\sim 1 \text{ MeV}$ )**
  - $\eta = 1 \times 10^{-6} \rightarrow \delta a_e \approx 1.05 \times 10^{-9}, \delta E_H \approx -2.3 \times 10^{-11} \text{ eV}, \delta E_{\mu H} \approx -2.6 \times 10^{-6} \text{ eV}.$

These are **order-of-magnitude** indicators: muonic shifts can be orders of magnitude larger than electronic ones for the same  $\eta$  and  $\Lambda$  because the muonic Bohr radius is much smaller.

---

## Updated, realistic sensitivity picture

- **Electron g–2** (using the numerical VP integral) gives **extremely strong limits** on  $\eta$  at low-to-moderate  $\Lambda$  where the VP insertion overlaps the electron mass scale. In the model used, for  $E$  roughly in the eV–keV range, an  $\eta$  of order  $1e-6$  produces  $\delta a_e$  comparable to the current  $a_e$  uncertainty; therefore  **$\eta$  values larger than  $\sim 10^{-7}$ – $10^{-9}$**  (scale dependent) are already excluded by electron g–2 in this toy model. The exact exclusion curve is in /mnt/data/updated\_eta\_exclusion.png. The numerical evaluation and the standard integral representation are the correct way to compute these bounds (replacing the previous crude scaling). See methodology references for VP insertion into a $l$ . ([arXiv](#), [Inspire](#))

- **Muonic hydrogen (Antognini 2013)** is highly sensitive to short-range structure but its **experimental uncertainty** ( $\Delta E \simeq 3.14 \times 10^{-6} \text{ eV}$ ) is much larger than tiny optical clock uncertainties. For many  $\Lambda$  values the muonic measurement places **weaker**  $\eta$  limits than electron  $g-2$  (because although muonic  $\delta E$  is large per unit  $\eta$ , the experimental  $\Delta E$  is still not tiny compared with the scale of  $\delta E$  for small  $\eta$ ). Still, muonic data are a **critical cross-check** because they probe different scales and can decisively amplify short-range deviations that leave  $a_e$  largely unaffected. ([PubMed](#))
- **Electronic hydrogen / optical spectroscopy** (1S–2S) has extremely small fractional uncertainty but its sensitivity to short-range vacuum effects is weak compared to muonic atoms (the Bohr radius is  $200\times$  larger), so the combined dataset must be fitted jointly to place the most robust constraints on  $(\eta, \Lambda)$ .

All these conclusions assume the **toy**  $\Pi$  model used here. A different spectral shape (e.g., a narrow resonance, a thermal-like distribution, or a multi-component  $\Delta\Pi$ ) can shift which experiment is most constraining.

---

## Citations (most load-bearing sources used in calculation & choices)

- On vacuum-polarization insertions and numerical techniques for their contributions to  $a_{\mu}^{\text{had, VP}}$ : e.g. detailed calculations and methodology in recent arXiv work on VP contributions and integrals. ([arXiv](#))
- Review/overview of theoretical status for  $a_e$  and how VP insertions appear in the theory & error budget. ([Inspire](#))
- Antognini *et al.*, Science 2013 — muonic hydrogen 2S–2P measured frequency 49,881.88(76) GHz ( $\Delta\nu=0.76 \text{ GHz} \rightarrow \Delta E \approx 3.14 \times 10^{-6} \text{ eV}$ ). I used this accurate experimental uncertainty in the updated exclusion curves. ([PubMed](#))

(If you want formal paper-style references / DOIs in the figure captions or later drafts I can put full citations into the notebook output.)

---

## Important caveats & where this is still approximate

1. **Toy  $\Pi$  model.** I used  $\Pi(-Q^2) = \eta \Lambda^2 Q^2 + \Lambda^2 \Pi(-Q^2) = \eta \Lambda^2 \frac{Q^2}{\Lambda^2 + Q^2}$ . That is a simple, smooth, single-scale model chosen to keep the mapping transparent. Real PD microphysics might produce multimodal spectra, sign-changes, or narrow features that change the relative sensitivity of  $g-2$  vs  $\Lambda$  shifts.

2. **Yukawa potential proxy for bound states.** For the Lamb shift I used  $\langle V_{\text{pip}} \rangle$  with the Yukawa spatial form that corresponds to the above  $\Pi$  in momentum space. That is a physically consistent toy but **not** the full Uehling potential derived from QED  $\Pi(q^2)$  including relativistic and recoil corrections. For publication-grade limits you should calculate the **full** Uehling-like correction produced by  $\Delta\Pi(q^2)$  and use Dirac wavefunctions (and radiative/recoil/nuclear corrections) for  $\mu\text{H}$  and  $\text{H}$ ; see the muonic Lamb-shift theory literature for the full set of corrections. ([mpq.mpg.de](http://mpq.mpg.de), [ScienceDirect](#))
  3. **Higher-order insertions & renormalization.** The calculation used the leading VP insertion into the one-loop vertex formula. For large  $\eta$  or exotic spectral shapes you would need to check higher-order insertions, mixing with hadronic-like pieces, and renormalization conventions; existing literature covers the rigorous techniques (dispersion relations, multi-loop integrals). ([arXiv](#))
  4. **Systematics vs theory uncertainties.** For hydrogen spectroscopy the dominant limitation for some comparisons is nuclear structure + proton radius; muonic experiments reduce some of those uncertainties but bring in their own nuclear corrections. Joint fits should treat nuclear-model nuisance parameters explicitly.
- 

## Recommended next steps (concrete)

If you want me to continue, I recommend this ordered plan and I can execute any item next:

1. **Full  $\Pi \rightarrow$  Uehling calculation.** Replace the Yukawa expectation with the **full Uehling potential** computed from the chosen  $\Pi(q^2)$  and evaluate  $\delta E$  using *relativistic Dirac* wavefunctions (especially important for muonic hydrogen). I can implement that numerically and replot limits. (This removes the main approximation in the Lamb-shift piece.)
2. **Refine  $\Pi$  model families.** Implement several physically motivated spectral shapes for  $\Delta\Pi$  (Lorentzian resonance, thermal/blackbody-like, multi-component), and produce combined exclusion contours in the  $(\eta, \Lambda)$  plane for each shape.
3. **Produce publishable tables.** Compute 95% CL exclusion curves (not just the  $\eta$  that equals uncertainty) by propagating experimental uncertainties and combining  $a_e + \mu\text{H} + \text{optical}$  data in a joint likelihood, including nuclear/finite-size nuisance parameters. I can produce CSVs and plots suitable for a preprint.
4. **Deliver raw data & code.** I can export the notebook, the numerical grids, and the exact code used so your experimental collaborators can reproduce and extend it.

Tell me which of the four you want next — I can start (1) the full Uehling calculation or (2) implement several  $\Pi$  shapes and rerun the numeric pipeline immediately.